# Electro-Optical Tracking Considerations II

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## ABSTRACT

This paper is a tutorial on factors affecting the detectability and tracking of targets using visible and IR sensor systems. Topics to be addressed are the effects associated with atmospheric attenuation, sensor system parameters, and target characteristics including reflectivity, size and range. Also considered are servo and optical system characteristics and their effects on tracking system performance. This paper complements a paper presented at Acquisition, Tracking and Pointing III, which addressed the performance and design parameters for electro-optical tracking systems.

**Keywords:** FLIR, servo, atmospheric attenuation, sensor, contrast, optics, MWIR, LWIR, spatial resolution

## 1. INTRODUCTION

An electro-optical tracking system can be viewed as a collection of interdependent subsystems and parameters, which must be analyzed in determining their effect on the overall performance of the integrated system. The approach presented in this paper looks at methods for analyzing each of the subsystems and parameters encountered in structuring an optimal system configuration. Target characteristics and environmental parameters[1] which must be considered in structuring the tracking system include the following:

- Size
- Range
- Dynamics
- Background Radiance
- Target Radiance
- Atmospheric Attenuation
- Spectral Region

Figure 1 illustrates the system model to be used in analyzing the various system elements.

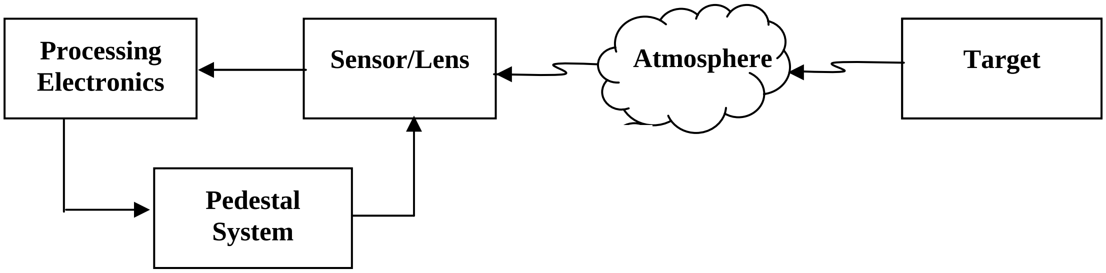

Figure 1 – System Model

### 1.1 Sensor system analysis (Visible)

The following section looks at the effects on pointing accuracy and target detectability based on the spatial characteristics of the target, optical system focal length and sensor characteristics. To demonstrate these effects a set of requirements listed in Table 1.1 will be used in illustrating a techniques for analyzing system performance.

Table 1 - Target Parameter

| Parameter | Minimum | Maximum |
|---|---|---|
| Range | 0.3 km | 31 km |
| Target Size | 1.0 meters | 10 meters |
| Optics (Focal Length) | 500 mm | 3000 mm |

**Sensor Format**: CCD 754/480 (h/v) elements, 2/3 inch format (8.8mm x 6.6mm)

The **angular field-of-view (FOV)** of the sensor and optical system can be derived from the following relationship:

Tan $\alpha/2$ = ½ (sensor linear dimension) / (focal length of the optics) or $\alpha = 2\text{Tan}^{-1}$ (linear dimension / 2 fl)

Where $\alpha$ = the sensors angular field-of-view. Table 1.2 shows the relationship between the sensor's linear dimensions and focal length (FL).

Table 2 - Sensor Field of View versus Focal Length

| Focal Length | Horizontal FOV mr /degree | Vertical FOV mr /degree |
|---|---|---|
| 500 mm | 17.6 mr / 1° | 13.6 mr / 0.8° |
| 1000 mm | 8.8 mr / 0.5° | 6.6 mr / 0.4° |
| 3000 mm | 2.2 mr / 0.13° | 1.75 mr / 0.1° |

If an alternative sensor format is selected the field of view of the system varies accordingly. Figure 2 represents the standard sensor formats available with common CCD cameras. Table 1.3 shows the effects of the senor size on the system's angular FOV.

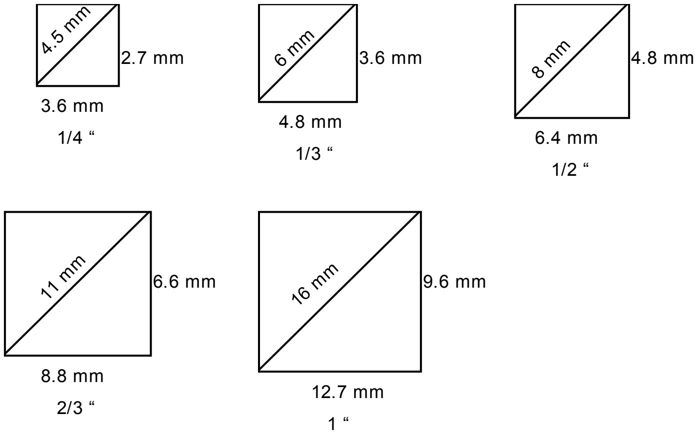

Figure 2 - Standard CCD Sensor Formats

| Sensor Format | Horizontal FOV 1000 mm fl | Vertical FOV 1000 mm fl |
|---|---|---|
| 1/4 ″ | 3.6 mr / 0.2° | 2.7 mr / 0.15° |
| 1/3 ″ | 4.8 mr / 0.28° | 3.6 mr / 0.2° |
| 1/2 ″ | 6.4 mr / 0.4° | 4.8 mr / 0.28° |
| 2/3 ″ | 8.8 mr / 0.5° | 6.6 mr / 0.38° |
| 1″ | 12.7 mr / 0.7° | 9.5 mr / 0.55° |

Table 3 - Sensor FOV versus Sensor Format Size

It should be noted that for applications requiring a wide dynamic range or where gray-level sensitivity is important, the size of the sensor requires consideration. As the size of the sensor cells decrease, the sensitivity of the sensor deceases. For most tracking applications a 1/2-inch or 2/3-inch format sensor is recommended.

Once the sensor's angular FOV has been determined, the next step in the analysis process is to determine the **sensor's angular resolution $S_\theta$.** This is also referred to as the sensor's instantaneous FOV (IFOV), and is given by the following relationships:

$S_{\theta\ \text{Horizontal}}$ = (Sensor Horizontal FOV) / (Number of Pixels/Line)

$S_{\theta\ \text{vertical}}$ = (Sensor Vertical FOV) / (Number of Lines /Field)

Based on the camera system parameters given of 754/480 (h/v). Table 1.4 illustrates the system's angular resolution as a function of focal length.

Table 4 - System Angular Resolution versus Focal Length

| Focal Length | Horizontal Resolution degrees /pixel | Vertical Resolution degrees /TV line |
|---|---|---|
| 500 mm | 0.014° | 0.003° |
| 1000 mm | 0.0007° | 0.0015° |
| 3000 mm | 0.00015° | 0.0004° |

With the sensor's angular resolution established, the next step is to determine the target's angular size as a function of range and minimum and maximum size. The **target's angular size** $\phi$ is derived from the following relationship:

Tan$\phi^{-1}_{\text{target}}$ = (Target Linear dimension) / (Range to Target)

Where $\phi$ = the target's angular size. Figure 3 illustrates this geometric relationship. For small values of $\phi$ the Tan$^{-1}$ is approximately equal to the target size in milliradians (mr). Table 1.5 illustrates the target's angular size for various ranges and target aspects.

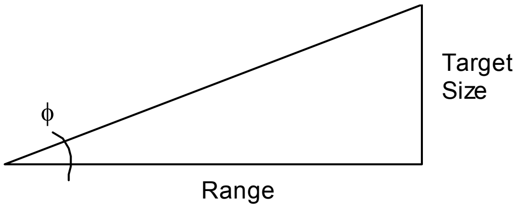

Figure 3 - Target Angular Size versus Range

Table 5 - Target's Angular Size versus Range/Size

| Range (km) | Target Size (meters) | Angular Size (mr /degrees) |
|---|---|---|
| 3 km | 1 m<br>10 m | 0.33 mr / 0.02°<br>3.3 mr / 0.2° |
| 15 km | 1 m<br>10 m | 0.07 mr / 0.004°<br>07 mr / 0.04° |
| 30 km | 1 m<br>10 m | 0.035 mr / 0.002°<br>0.35 mr / 0.02° |

The final step in the analysis determines the number of pixels and TV lines the target represents to the sensor (tracker) at various ranges and aspects. The following relationship maps the targets angular size to lines and pixels in the sensor's FOV.

The number of **target pixels/TV lines = (Target Angular Size) / (Sensor Angular Resolution pixels / TV lines)**. Table 6 shows the target's size in pixels and TV lines as a function of range and aspect.

Table 6 - Target Size (pixels/lines) as a Function of Range/Size

| Range (km) | Target Size (meters) | Target Size (pixels) | Target Size (TV lines) |
|---|---|---|---|
| 3 km | 1 m | 14 | 6.6 |
| | 10 m | 140 | 66 |
| 15 km | 1 m | 3 | 1.2 |
| | 10 m | 32 | 13 |
| 30 km | 1 m | 1.4 | -- |
| | 10 m | 14 | 6.3 |

The above table does not address the effects of atmospheric extinction, sensor spectral characteristics, sensitivity or the modulation transfer function (MTF) of the system on target detectability. The following criteria have been established for a human observer[2].

Target detection - 1.0 line pair; Recognition - 4.0 line pair; Identification - 6.5 line pair

## 1.2 Sensor system analysis (Infrared)

FLIRs typically operate in two spectral bands, the 3-5 microns and 8-14 microns, while the human eye is sensitive to energy in the 0.3-0.6µ region. The 3-5µ region is referred to as the Mid-wave Infrared (MWIR), with the 8-14µ region referred to as the Long-wave Infrared (LWIR). Other infrared bands include the Very Long-wave Infrared (VLWIR) from 14µ to 25µ , Far-wave Infrared (FWIR) from 25µ to 100µ and the Extreme-wave Infrared from 100µ to 1000µ . Figure 4 shows the transmission characteristics of the atmosphere for a one nautical mile sea level path.

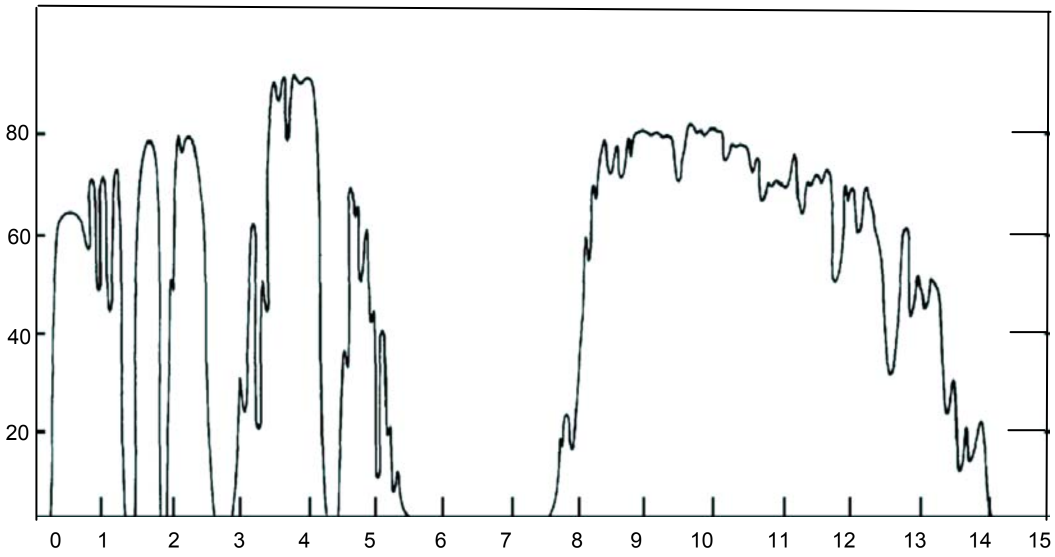

Figure 4 - Atmospheric Transmission Characteristics Infrared Region

As shown in Figure 4 the atmosphere provides windows allowing transmission of the three spectral bands, Visible, MWIR and LWIR with minimal attenuation[3]. For most other wavelengths, the atmosphere is relatively opaque. A fourth window present in the atmosphere is in the Near Infrared (NIR) region from 0.9µ to 1.7µ. In some applications, by using a CCD sensor having extended red (0.7µ to 1.0µ) sensitivity, improved atmospheric penetration can be achieved.

Energy emitted in the visible spectrum results from reflections of natural (sun light) or artificial illumination from the surface of an object. Without a source of illumination, an object cannot be detected using a CCD camera. The 3-5µ region contains energy that is a result of sunlight (reflected energy) plus thermal energy (radiated energy) from the target. The most noticeable aspect is the appearance of shadows during daylight that is not present in the 8-14µ region. Energy emitted in 8-14µ region is the result of thermal energy present in the object.

Thermal imagers are passive sensors that need no source of radiation such as sunlight. It should be noted reflections do occur on materials such as glass or water.

Solar reflectance in the 3-5µ region can be an issue during daylight imaging of an object. In viewing a target on water, glint from waves is more prevalent in the 3-5µ region making it unusable during daylight hours for maritime operations. 3-5µ sensors are used in surveillance applications since lettering or painting is visible in this region, but is generally absent from 8-14µ. This is an important requirement for identifying a ship or airplane in law enforcement and surveillance applications.

In observing effects of the atmosphere on the transmission of IR energy, there are two major differences between MWIR and LWIR bands. The 3-5µ wavelength has better transmission characteristics in atmospheres with high concentrations of water vapor and aerosols, which tend to absorb 8-14µ energy at a higher rate. Therefore, FLIRs operating in maritime or in regions with high water vapor content operate better in the MWIR region. Tactical systems, which typically operate in smoke and dust conditions, operate better in the LWIR region since 3-5µ energy is absorbed by smoke and dust[3].

Another consideration in selecting LWIR versus MWIR is the amount of thermal energy generated by the object. The lower the temperature, the more the 8-14µ band is favored because the limited amount of energy available in the 3-5µ band. For operations in cold, winter environments, an 8-14µ FLIR will provide better performance. In hot climates, especially with high water vapor and aerosol content, a 3-5µ system is better suited. Another consideration in the selection of a FLIR is the range to target. For target ranges less than 10km, 3-5µ and 8-14µ sensors work equally as well. For target ranges greater than 10km, a MWIR system will normally provide better performance over LWIR systems. However, a sensor operating in the 8-14µ range typically provides better detectability for small ΔT targets against a constant background.

### 1.2.1 Thermal resolution

For FLIR systems an important measure of performance is ability to detect small changes in temperature. The smallest temperature difference a system can detect is called the thermal resolution. Changes equal to or less than the system background noise will not be detected. Thermal resolution can be described by NETD (noise-equivalent-temperature-difference)[3]. NETD is the temperature change, which changes the collected flux by an amount equal to the noise-equivalent-power (NEP).

The thermal resolution or NETD can be improved by increasing the area of the detecting elements. This allows more energy to be collected by each detector element. By increasing the size of the detector elements the sensor's spatial resolution is degraded by an increased IFOV. The thermal and spatial resolution of FLIR sensors is, inversely proportional.

### 1.2.2 Spatial and Thermal resolution, MRTD

With current sensor technology it is not possible to achieve high spatial and thermal resolution, although this is changing with newer sensors and techniques such as dithering. Used independently neither parameter is a good measure of the overall FLR system performance. A single quantity, called the minimum resolvable temperature difference, MRTD measures both performance parameters simultaneously. MRTD is determined experimentally and therefore takes into account all of the various theoretical and real-world factors of importance. Determination of the MRTD is done by slowly heating a bar pattern test target at some range from the detector. The four bar test target used to determine MRTD is shown Figure 5.

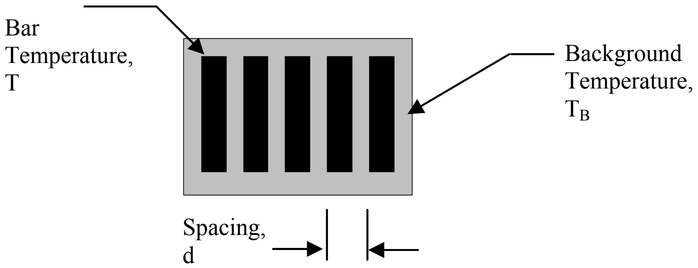

Figure 5 - MRTD Target

The spacing d represents a single cycle of the test pattern. The spatial frequency is given by 1/d with units of cycles/m. Since the spatial extent is related to the IFOV by the range, the spatial frequency can be expressed as cycles/mr calculated from 1000/(R d). Where R is the distance to the target and d is the spacing between the bars

MRTD is the temperature difference at which bars first become visible against the background. MRTD has units of °C at a given spatial frequency (in cycles/mr). MRTD combines both spatial and thermal resolution into a single quantity that can be used to compare the performances of FLIR systems.

As an example: If the sensor's MRTD = 0.05°C at 0.5 cycles/mr, determine the sensor's thermal and spatial resolution at a range of 1km.

The thermal resolution is 0.05°C, which represents the smallest temperature change that can be detected at any range.

The spatial resolution can be calculated as follows. First compute the IFOV which is given by:

IFOV = 1/ spatial frequency = 1/0.5 cycles/mr = 2 mr.

Therefore, at R = 1km, the sensor's spatial resolution is
Spatial Resolution = R x IFOV = (1km) x (0.002 radians) = 2 meters

## 1.3 Lens considerations

The diversity of lens available for tracking and surveillance applications range from simple fixed focal length optics to multiple FOV and zoom optical systems. Considerations for lens systems encompass a range of parameters[1] including:

- Focal Length
- Spectral Characteristics
- Aperture
- Lens Distortion
- Atmospheric Effects
- Boresight Shift
- Optical Distortion
- Zoom Feedback

## 1.4 Sensor considerations

The following summarizes some of the key characteristics[1], which need to be considered in selecting a sensor system.

- Spectral characteristics
- Blemishes
- Resolution
- Noise
- Lag
- Sensitivity
- Linearity
- Blooming

## 1.5 Target contrast

The U.S. Air Force Handbook of Geophysics gives the irradiance from the sun outside the atmosphere as 1,396.4 watts/cm$^2$. If this is integrated over the luminosity curve, the illuminance is found to be 12,551 foot candles. The direct illuminance from the sun is attenuated about 30 percent passing through the atmosphere to the surface of the earth. Neglecting the altitude difference between the sensor and object, this leaves about 9,000 foot candles as the luminance falling on the object indirect sunlight. If the object observed has a reflectivity of 50% (as does aluminum or hastelloy aircraft skin), the luminance of the object is 4,500 foot lamberts. The target contrast is then:

$C_0 = |B_b - B_t| / B_b$ where $B_t =$ Target Luminance, $B_b =$ Background Luminance

$C_0 = |9{,}000 - 4{,}5000| / 9{,}000 = 50\%$ the inherent contrast

The apparent brightness of the target at the sensor is the target brightness attenuated by the transmission through the atmosphere. This then becomes:

$$C_R = e^{-\sigma R} [\,|B_b - B_t| / B_b]$$

Where $e^{-\sigma R} = T_a$ (atmospheric path transmittance)$^2$ at visible wavelengths where molecular and aerosol absorption are negligible.

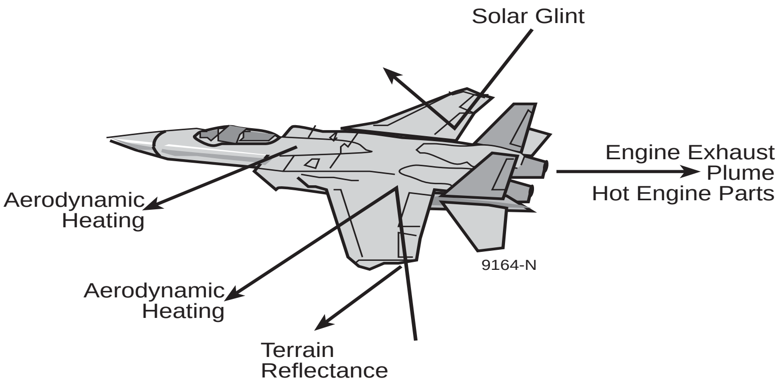

Figure 6 – Target Contrast Model

### 1.5.1 Visual contrast analysis

If a source emits radiation, the irradiance at some distance from the source is calculated according to the inverse square rule if the path is through a vacuum. However, if the path is through a gaseous atmosphere, some of the radiation is absorbed and scattered to modify the received irradiance by some transmission factor. Atmospheric transmittance is a function of many variables, wavelength, path length, atmospheric gases, pressure, temperature, fog, dust, aerosols, and the size of their particles.

For a target of 50% contrast at range R, we will consider the atmospheric extinction as this factor will limit the perceived contrast. This extinction factor is related to the transmission of a given optical path by the relationship:

$T_a = e^{-\sigma R}$
$R =$ optical path length (km)
$\sigma =$ extinction coefficient (km$^{-1}$)

The meteorological range or “visibility” is defined for a given horizontal range R for which the transmission of the atmosphere in daylight reduces an object’s contrast to 2% ($C_R/C_0$). This apparent contrast $C_R$ reduces exponentially with range R according to $e^{-\sigma R}$ and is depicted in Figure 7.

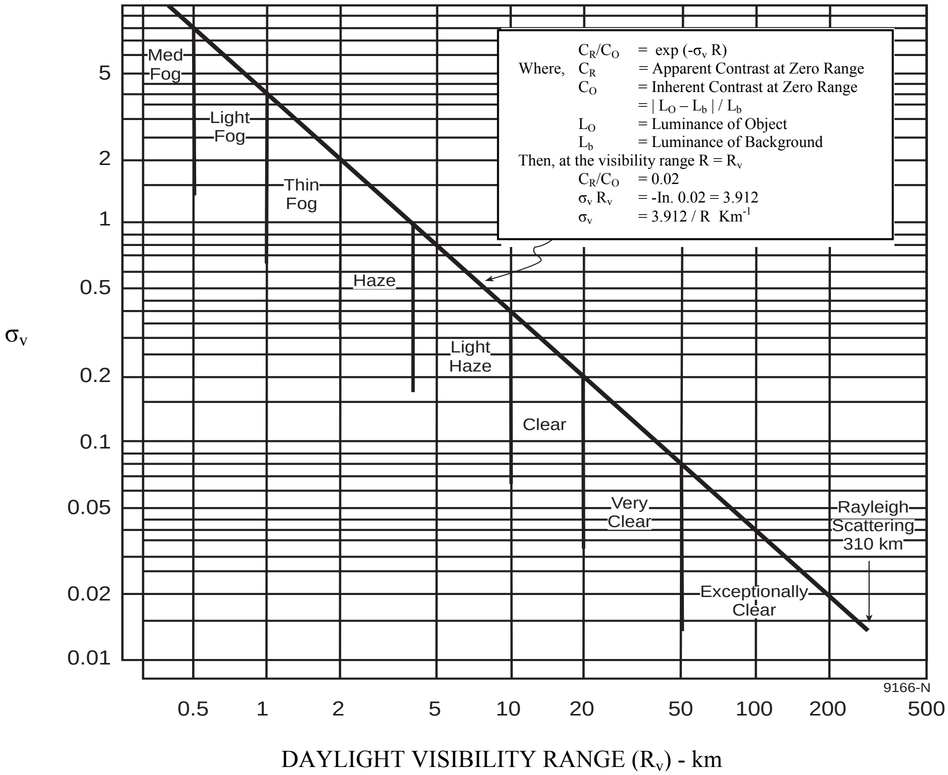

Figure 7 - Atmospheric Attenuation Coefficient for Visible Light

A wavelength of 0.55 microns is selected as a “typical” wavelength at or near the peak response wavelength of the sensor for purposes of transmittance calculation. It has a further advantage in that it is the center of the 0.4 to 0.7 micron visible wavelength band. This then gives the following total attenuation$^2$ (scattering coefficients ($\sigma_t = \sigma_a + \sigma_m$) at sea level and various horizontal layers at altitude 3-4 km and 4-5 km as shown in Table 7.

Table 7 - Total Attenuation at Sea Level

| **Visual Range** | **Sea Level** | **Layer 3-4 km** | **Layer 4-5 km** |
|---|---|---|---|
| 5 km | 0.7824 | 0.0307 | 0.0156 |
| 10.8 km | 0.3622 | 0.0226 | 0.0141 |
| 23 km | 0.1701 | 0.0172 | 0.0131 |
| 40 km | 0.978 | 0.0157 | 0.0128 |

If the initial A/C acquisition range is 18km at an elevation angle of 35°. The atmospheric extinction reduces as the elevation angle (altitude) to the observed object increases. This increase in transparency of the atmosphere is due to the lessening of the optical density or atmosphere between the object and the observer. Data suggests that for objects above the 6km layer that a modifier of 0.2 is used for the extinction coefficient.

The Table 8 provides a correction factor for σ as a function of altitude[2]. The increased "seeing" conditions occur as the optical density lessens as one views the target at increasing elevation angles and is related to the secant of that angle. Figure 8 shows the attenuation coefficient for slant and horizontal path lengths.

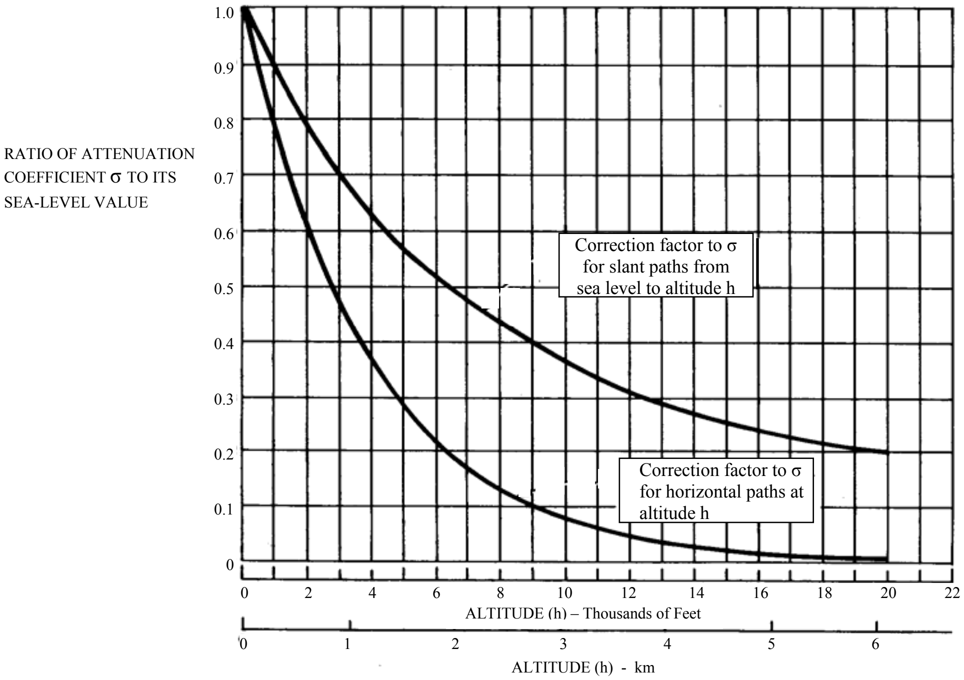

Figure 8 - Atmospheric Attenuation for Slant and Horizontal Path Lengths

Table 8 Correction Factor for σ as a Function of Altitude

| **Altitude (km)** | **Slant Path (Correction Factor)** |
|---|---|
| 1.0 | 0.68 |
| 2.0 | 0.50 |
| 3.0 | 0.38 |
| 4.0 | 0.29 |
| 5.0 | 0.24 |
| ≥6.0 | 0.20 |

Then for an acquisition range of 18km, the contrast at the sensor would be:

Standard Clear Atmosphere: $C_R = e^{-(0.17)(0.2)(18)}(0.5) = 27\%$

Light Haze Atmosphere: $C_R = e^{-(0.36)(0.2)(18)}(0.5) = 14\%$

For the stated condition, the target contrast is more than sufficient for effective auto-track. For high performance trackers, a target contrast of 2-3% is adequate for tracking.

## 1.6 Tracking techniques

There are a number of techniques that can be implemented to follow the motion of a target of interest. These techniques fall into two general categories, Open-Loop Tracking and Closed-Loop Tracking.

### 1.6.1 Open-Loop tracking

**Manual Only track** – the Manual Only tracking technique requires an operator to view the target at all times. He views the target either through a sighting telescope or by means of a TV monitor. The quality and the accuracy of the mission data depends solely upon the skill of the operator in keeping the target within the field-of-view of the optical system.

The targets that are trackable with the Manual Only tracker are as follows:

Low speed missiles or long-range tracking of medium speed missiles
Non-evasive slow moving aircraft

The targets that are not trackable by the Manual Only tracking technique are as follows:

High speed missiles
High speed aircraft
Medium speed maneuvering aircraft

**TSPI and Slave tracking** – The TSPI and Slave Tracking technique is similar to the Manual Only track with the single exception that remote radar, aircraft TSPI or other external source can slave the tracking mount (TM) to the target. This is helpful when the target is obscured by clouds or backgrounds, or when the vehicle is moving too fast for the operator to keep it reliably in his field-of-view. No accurate real-time data is available from the optical tracking system with a manual track. The targets that are trackable with an external positioning source slave track mode are as follows:

High speed aircraft
Medium speed missiles after launch
The same targets as described in the preceding paragraph on Manual Only track

The following targets can be difficult to track using TSPI or radar slave tracking techniques:

High speed missiles from launch
Targets against a terrain background (radar)
Targets with a low radar signature profile (radar)

**Trajectory tracking** – Trajectory tracking uses the tracking site computer to predict the location of the target in space. The computer then drives the tracking mount to this position in space and continually keeps the tracking mount on the predicted trajectory. This type of tracking system works well on a missile launch if the missile or aircraft follows the predicted trajectory and has no malfunctions or deviations during flight.

**1.6.2 Closed-Loop tracking** - The types of trackers listed below fall into the category of closed loop automatic trackers. These trackers generate X-Y error signals, which are used to close the loop on a pedestal system keeping the target positioned at the system line-of-sight (LOS)[1,4].

**TV Trackers** – Three techniques commonly used in tracking systems to determine the target's angular errors are, Edge, Centroid and Correlation[4] tracking. These methods typically employ a tracking gate to reject a major portion of the field-of-view and allow processing of only the selected track space point area. Thus, the gate may be used to reject unwanted background signals .

*Edge Tracking* - A tracker may employ a simple edge tracking technique allowing the leftmost, rightmost, leading or trailing edge of the target to be tracked. This allows the track point of interest to be followed even though the target attitude changes. This is extremely important if a missile's attitude changes during its trajectory; for instance, during launch the leading edge of the target would be tracked and as the target changes its attitude to go in a more horizontal direction, either the left or the right edge may be selected and the track point will remain on the nose of the vehicle. This feature can be used to detect aircraft missile operation and allow the system to automatically switch from tracking the aircraft to tracking the missile. Leading Edge Track is another algorithm which automatically follows the leading edge of the target based on direction vectors generated from the mount encoder data.

*Centroid Tracking* - In addition to the multiple edge tracking capability, other algorithms typically employed are centroid tracking techniques: Binary Mass Centroid, Gray Level Mass Centroid, and Intensity Centroid. The centroid tracking algorithms integrate the video within the gate by partitioning the active gate area into a matrix. Each cross point in the matrix is a specific horizontal and vertical coordinate of the digitized video. The processor integrates the horizontal and vertical coordinate data of the digitized video signal and derives the target centroid information. The resulting track point indicates the true center of mass of the target and allows tracking low contrast targets.

*Correlation Tracking* – Correlation tracking systems use a pattern matching technique in tracking the target against a cluttered background or tracking a specific point on the target of interest. The ability to track a specific point on the target will allow the operator to track the missile mounting point on the aircraft for reliable detection of the missile release. This mode coupled with a second gate will allow smooth transition from aircraft to missile track.

For tracking applications the tracker's sampling rate should be equal or greater than the video bandwidth of the sensor. Most high performance trackers can easily accommodate target size of 2-pixels by 2-TV lines at an SNR of 2:1. Single pixel targets are trackable but typically require a higher SNR, on the order of 5:1. These SNR numbers are based on limited preprocessing of the video information. A parameter used as a measure of tracker performance is the target's contrast ratio $C_r$ given by the following relationship:

$C_t = (|C_b - C_t|) / (C_t + C_b)$ where $C_t$ is the target luminance and $C_b$ the background luminance

Another factor which needs consideration in selecting a tracker is the amount of latency in outputting the error signals. This latency can be the result of preprocessing and/or frame buffering before the target coordinates are calculated and transmitted. In using serial communications links, baud rate can also be a factor in introducing latency in the output errors, especially when tracking high dynamic targets.

## 1.7 Servo system analysis

The bandwidth of a servo system is directly related to the speed of response via the relation $2\pi(f)T = 1$ where T = system time constant. The speed of response is of importance during target acquisition. In order to illustrate this, consider a tracking system for airborne targets with a total field of view of 2A in one dimension. Now imagine the target of interest is moving at a uniform angular rate ($\dot{\theta}$) with respect to the tracking site. The tracking mount is at rest; acquisition is to be accomplished by switching the track system to "automatic" while the target is in the field of view. Referring to the Figure 9 below, it is obvious that if the target is allowed to reach +A, the edge of the field, before automatic track is attempted, then the target will escape; acquisition will not be successful.

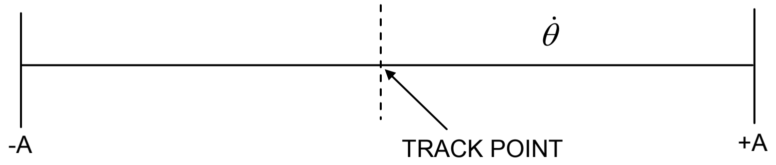

Figure 9 - Acquisition Geometry

Whether or not acquisition will be successful when the target is deeper in the field of view depends, in general, on the value of target angular velocity, the system bandwidth, loop gain, and damping factor. Thus, the effect of reduction of bandwidth is to make the field of view of the system smaller for acquisition of moving targets. The effect is not linear, however. The acceleration of the tracking mount increases as the square of the first corner frequency, other factors being equal. Thus, it can be seen that the ease of acquisition of a high speed target is quickly enhanced by increased bandwidth.

According to the foregoing, it would appear to be desirable to design servo elements with infinite bandwidths. This would give ideal correspondence between input and output. In practical cases, however, the servo does not want to follow all inputs. It is desirable to follow only those inputs that are due to noise or output disturbances should be rejected by the servo loop. Thus, the error signal spectrum must be filtered to attenuate system noise. In addition, the sensitivity of the transducer is often a function of bandwidth. This is quite apparent in the case of imaging tracking units. It is then advantageous to limit bandwidth to increase sensitivity.

It has been shown above that the choice of bandwidth for a servo gain is influenced by five factors: accuracy, target acquisition, track noise, and transducer sensitivity. The relative importance of these five variables is determined by the specific control problem. For a given problem, a compromise bandwidth must be selected from consideration of all five factors in order to ensure that the servo element will perform satisfactorily. In particular, it should be noted that the effect of bandwidth on accuracy is accumulative. That is, the error due to two elements of time constant T sec is twice the error resulting from a single element of time constant T. The same is true with regard to stability considerations; the effect of time constants is also accumulative. This, of course, is not the case with noise. Thus, each factor must be considered separately in order to ensure that the servo element is properly designed.

### 1.7.1 Tracking problem

To develop an appreciation for the effects of gain with respect to system error, we shall consider the following specific pass-course problem. Assume the target flies by at a constant velocity and altitude given below:

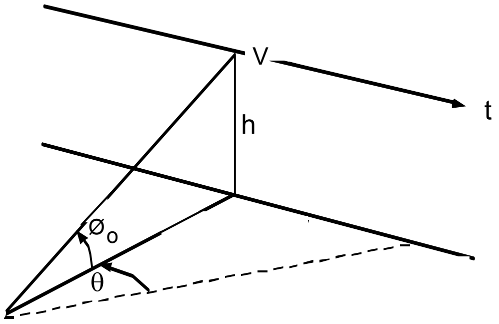

Figure 10 - Pass-Course Tracking Model

where: V = Target speed in meters/sec
h = Target altitude in meters
R = Range to target in meters
$\phi$ = Target elevation angle in degrees
$\theta$ = Target bearing angle in degrees ($\theta$ = 0° at crossover)

Successive differentiation obtains:

$$\dot{\theta} = [KV / Ro \cos \phi_0] \cos^2 \theta$$

$$\ddot{\theta} = [KV / Ro \cos \phi_0] \cos^2 \theta \sin 2\theta$$

The maximum values are found by setting the next higher derivative to zero and using the value of $\theta$ found. The maximum values are utilized to quickly determine the dynamic tracking errors of a specific system related to the feasibility of satisfying the stated problem.

$$\dot{\theta}_{max} = 57.3 \; [V / Ro \cos \phi_0]$$

$$\ddot{\theta}_{max} = 37.2 \; [V / Ro \cos \phi_0]^2$$

If we then take the example of the target at the closest edge of the tracking corridor:

| | |
|---|---|
| Ground Range: | 2 km |
| Altitude: | 5 km |
| Speed: | MACH I |

Then:

$V_t$ = 332 m/sec

Ro = 385 meters

$\cos \phi_o$ = 0.37

Therefore:

$$\dot{\theta}_{max} = 57.3 \; [332 / (5385)(0.37)] \approx 9.5° / sec$$

$$\ddot{\theta}_{max} = 37.2 \; [332 / (5385)(0.37)]^2 \approx 1.03° / sec^2$$

Qualitatively, the azimuth rate ( $\dot{\theta}$ ) is a time dependent function having the appearance shown in Figure 11.

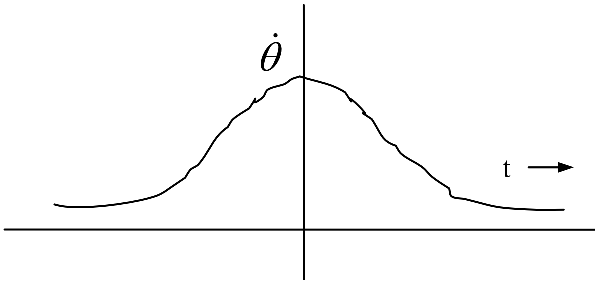

Figure 11 - Target Velocity Profile

The peak velocity occurs at the point of minimum range from the track site. The acceleration profile is illustrated in Figure 12.

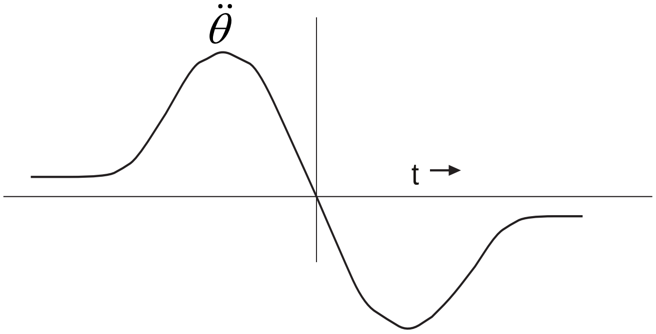

Figure 12 - Target Acceleration Profile

The peak accelerations occur at the inflection points of the velocity curve on either side of the minimum range point. The above figures encompass both acceleration and velocity to compute the combined effects. The total dynamic tracking error for a Type 1 servo system is approximated by:

$$e_t = e_v + e_a$$

where:

$$e_v = [\dot{\theta}_{max} / K_v]$$

$$e_a = [\ddot{\theta}_{max} / K_a]$$

Use the assumption that the $K_v = 500$ and as $K_a = 50$ for the mount, the dynamic tracking error is given by:

$$e_t = 9.5° \text{ sec} / 500 + 1.03° \text{ sec}^2 / 50$$

where:

$$e_t \leq 0.68 \; mr$$

This then sets the minimum parameters of the servo gain to obtain an acceptable track response and keep the target within the boresight axes.

Working backwards and assume the target accelerations stated below and mount a $K_a$ of 225 pointing error can be derived:

$$K_a = \dot{\theta} / e_a$$

$$e_a = \ddot{\theta} / K_a$$

$$e_a = 1.03° \text{ sec}^2 / 225$$

$$e_a = 0.08 \; mr$$

This would be adequate for accurately pointing longer focal length optics.

An optimized approach for the servo system configuration is to have a system that can adapt from a Type I to a Type II configuration. For initial acquisition of the target, the system is a Type I, providing a faster response for acquisition. The system then transitions to a Type II based on target position relative to bore sight or a timed interval.

### 1.7. 2 Pedestal considerations

The following parameters and characteristics[1] require consideration when configuring a pedestal system.

- Payload Configuration
- Azimuth & Elevation Travel
- Direct Drive or Gear Drive
- Velocity & Acceleration Profiles
- Pedestal Accuracy
- Encoder Type
- Wind Loading
- Encoder Resolution
- Type I or Type II Servo

### Summary

This paper has presented a set of basic tools and concepts for analyzing the various elements and effects typically encountered in configuring an Electro-Optical Tracking system. In determining an optimal system configuration each subsystem and effect requires analysis. These system elements include: the sensor and optics, target characteristics (size, range, reflectivity, spectral characteristics), atmospheric effects, operator interface, tracker functionality and pedestal performance. If these elements are thoroughly analyzed, the required system performance will be realized.

## References

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